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THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

VOLUME I

GENERAL

GENERAL DYNAMICS
Convair Division

REPORT NO. GDC-DDG-67-006

THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

VOLUME I

GENERAL

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GDC-DDG-67-006

THE STABILITY OF ECCENTRICALLY STIFFENED CIRCULAR CYLINDERS

VOLUME I

GENERAL

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ABSTRACT

This is the first of six volumes, each bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The complete set of documents was prepared under NASA Contract NAS8-11181 and furnishes workable design and analysis methods for the prediction of instability in such structures. The overall multiple-volume report includes design curves, procedures, and digital computer programs for the most important buckling modes. The material presented in this first volume provides an introductory background which puts the subsequent volumes in a proper perspective.

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DEFINITION OF SYMBOLS

Symbol	Definition
$A_{11}, A_{22}, A_{12}, A_{21}, A_{33}$	Elastic constants [see equations (3-4)].
$C_{11}, C_{22}, C_{12}, C_{21}$	Eccentricity coupling constants [see equations (3-4)].
$D_{11}, D_{22}, D_{12}, D_{21}, D_{33}$	Elastic constants [see equations (3-4)].
E	Young's modulus.
G	Modulus of elasticity in shear.
h	Distance between middle surfaces of sandwich facings.
L	Overall length of cylinder.
M_x	Running bending moment with respect to middle surface of basic cylindrical skin (acting on sections obtained by passing planes normal to the axis of revolution).
M_y	Running bending moment with respect to middle surface of basic cylindrical skin (acting on sections which lie in radial planes).
M_{xy}	Running twisting moment with respect to middle surface of basic cylindrical skin.
m	Number of axial half-waves in buckle pattern.
$\overline{N}_{THIEL}$	Loading parameter defined in equations (3-3), [positive for tensile loading].
$\left(\overline{N}_{THIEL}\right)_c$	$= -\overline{N}_{THIEL}$, [positive for compressive loading].
$\overline{N}_x$	Applied longitudinal tensile running load acting at the centroid of the effective skin-stringer combination.
$\overline{N}_y$	Circumferential tensile running load acting at the centroid of the effective skin-ring combination.
N_{xy}	Running shear load acting in the middle surface of the basic cylindrical skin.

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DEFINITION OF SYMBOLS
(Continued)

Symbol	Definition
n	Number of circumferential full waves in buckle pattern.
R	Radius to middle surface of basic cylindrical skin.
t	Thickness of basic cylindrical skin.
t_f	Sandwich facing thickness.
u, v, w	Reference-surface displacements (see Figure 5).
x, y, z	Coordinates (see Figure 5).
Z	Parameter defined by equation (3-2).
α	Parameter defined in equations (3-3).
β	Parameter defined in equations (3-3).
γ	Parameter defined in equations (3-3).
γ_{xy}	In-surface shear strain for middle surface of basic cylindrical skin.
γ_{xz}, γ_{yz}	Shear strains in planes normal to the middle surface of basic cylindrical skin.
ϵ_x	Longitudinal extensional strain for middle surface of basic cylindrical skin (positive for pure tension).
ϵ_y	Circumferential extensional strain for middle surface of basic cylindrical skin (positive for pure tension).
η_p	Parameter defined in equations (3-3).
η_s	Parameter defined in equations (3-3).
ν	Poisson's ratio.

GLOSSARY

Note: This glossary is meant to apply to all six volumes of the report. Separate glossaries are not included in each of the individual volumes.

Buckling of Isotropic Skin Panel - The initial buckling of the basic cylindrical skin whose boundaries are formed by the longitudinal and/or circumferential stiffeners. Buckling of the wall of unstiffened cylinders is a special instance of this mode of instability.

Local Buckling of Longitudinal Stiffener (Stringer) - The initial buckling of any leg or arc length of the cross-sectional shape of a longitudinal stiffener (stringer). Initial buckling of the outstanding flange of a Z-section stringer is an example of this mode of instability.

Crippling of Longitudinal Stiffener (Stringer) - The final ultimate compressive failure of a longitudinal stiffener which has a shaped cross section and is given sufficient support to prevent panel instability (see definition below). The crippling stress is the ultimate average stress for such a stringer.

Panel Instability - This mode of instability manifests itself as a bowing of the longitudinal stiffeners (stringers) into one or more axial half-waves without any radial displacement of the circumferential stiffeners (rings). Hence the axial half-wavelength cannot exceed the spacing between rings. Figure 1 depicts the special case where there is one half-wave between adjacent rings. Although not frequently encountered in practical structures, this mode can involve more than one such half-wave per ring spacing. Panel instability may or may not be preceded by buckling of the isotropic skin panels and/or local buckling of the stringers. The identification "Panel Instability" is somewhat of a misnomer since this terminology could easily lead one to the erroneous conclusion that reference is being

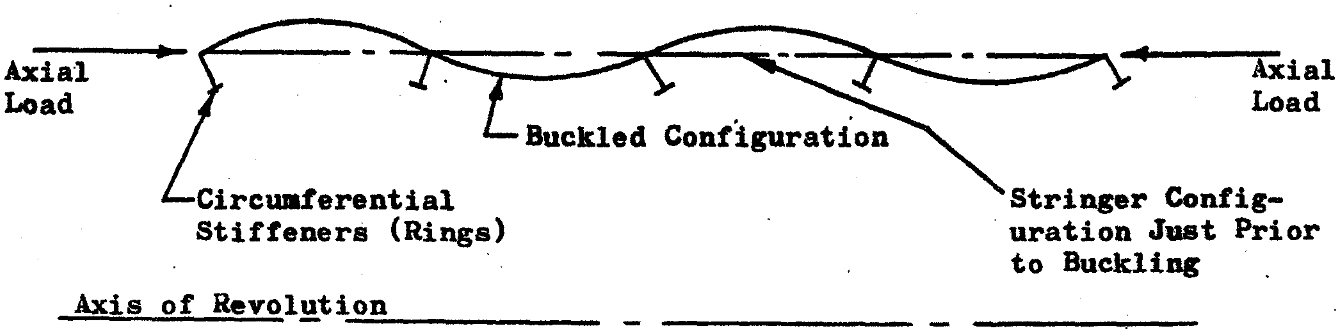

Figure 1 - Panel Instability

made to the mode which is identified above as "Buckling of Isotropic Skin Panel." A more suitable title could be selected but in the interest of maintaining consistency with the nomenclature usually found in the literature, the "Panel Instability" label has been retained in this study.

General Instability - This mode of instability involves the simultaneous radial displacement of both the longitudinal and circumferential stiffeners (stringers and rings). As shown in Figure 2, the axial half-wavelength

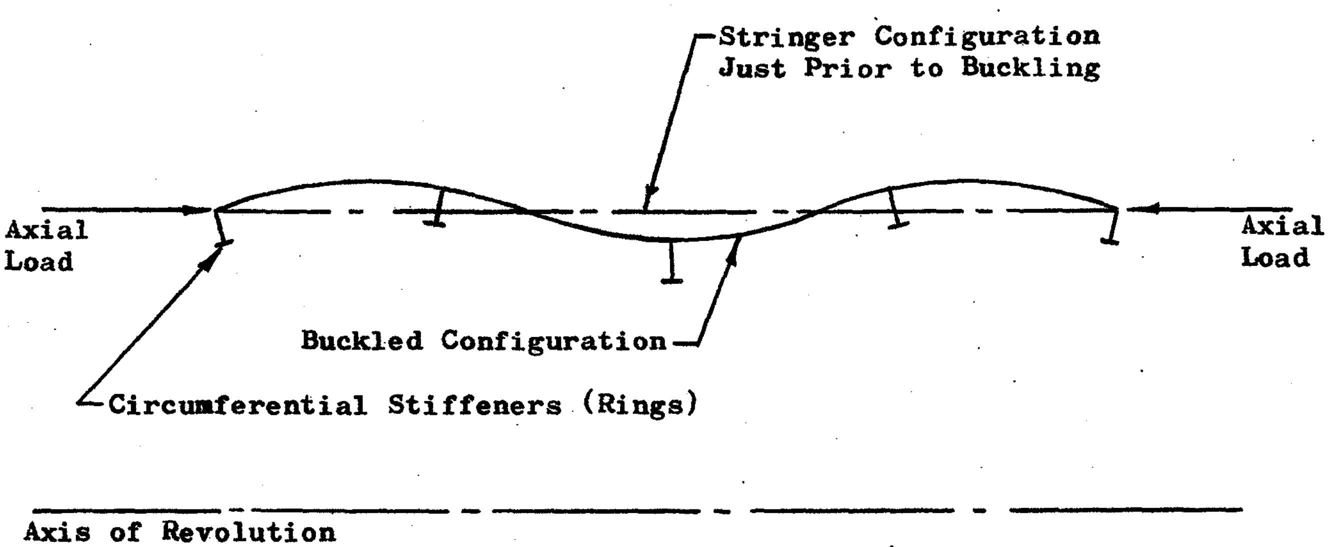

Figure 2 - General Instability

of the buckle pattern exceeds the spacing between rings. This phenomenon may or may not be preceded by buckling of the isotropic skin panels and/or local buckling of the stringers.

Stiffener Eccentricity - The distance from the reference surface to the centroid of the appropriate skin-stringer combination. Throughout this study, the reference surface has been taken as the middle surface of the basic cylindrical skin. Inside versus outside positioning of the stiffeners is identified by means of an appropriate sign convention.

Monocoque - This term comes from the French word meaning "shell only" and is used here to identify those configurations which do not incorporate any stiffening members (integral or non-integral). Note, however, that a monocoque configuration can have orthotropic properties.

Knock-down Factor - An empirical correction factor which is used in the reduction of classical small-deflection predictions to safe design levels. This factor is introduced primarily to allow for the detrimental effects from initial imperfections.

Stress Resultants - The six quantities N_x, N_y, N_{xy}, N_{yx}, Q_x, and Q_y obtained by integration of the infinitesimal loads over the shell wall (including skin and stiffeners), and the four quantities M_x, M_y, M_{xy}, and M_{yx} obtained by integration over the shell wall (including skin and stiffeners) of the infinitesimal moments with respect to a selected surface. The force stress resultants are expressed in units of force per unit length (lbs/in for example) while the moment stress resultants are expressed in units of moment per unit length (in-lbs/in for example).

Shell (or Shell Wall) - Throughout the several volumes of this report, repeated use is made of the terms "shell" and "shell wall". These terms are used interchangeably and are not meant to refer only to the basic cylindrical skin of the stiffened structure. They refer to the effective skin-stiffener combination. Whenever it is desired that reference be made solely to the basic monocoque cylinder which the stiffeners augment, the word "skin" will actually be included in the identification.

Anticlastic Bending - Bending into a deflected shape for which the principal radii of curvature have opposite signs. Bending of an initially flat plate into a saddle shape is an example. In addition, for beams, the Poisson-ratio effect results in anticlastic bending as depicted in Figure 3.

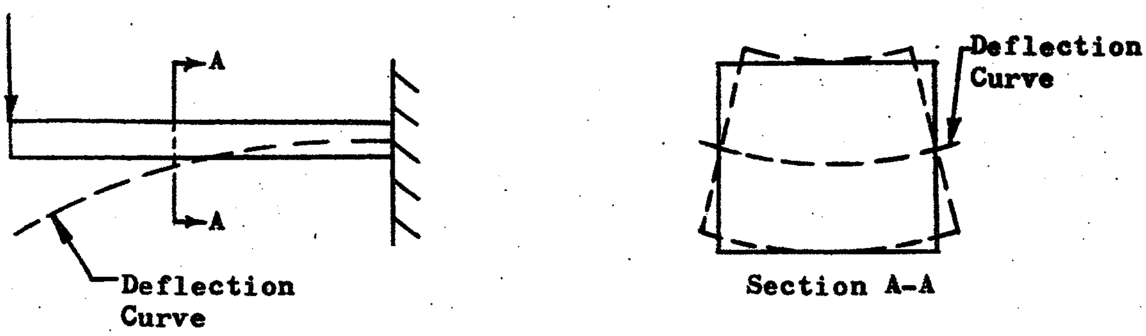

Figure 3 - Anticlastic Bending of a Beam

SECTION 1

INTRODUCTION

This is the first of six volumes, each bearing the same report number, but dealing with separate problem areas concerning the stability of eccentrically stiffened circular cylinders. The areas treated by the complete set of documents can be identified from the following listing of titles:

VOLUME I -	GENERAL
VOLUME II -	BUCKLING OF CURVED ISOTROPIC SKIN PANELS; AXIAL COMPRESSION
VOLUME III -	BUCKLING OF LONGITUDINALLY STIFFENED CYLINDERS; AXIAL COMPRESSION
VOLUME IV -	GENERAL INSTABILITY OF CYLINDERS HAVING LONGITUDINAL AND CIRCUMFERENTIAL STIFFENERS; AXIAL COMPRESSION
VOLUME V -	EFFECTS OF INITIAL IMPERFECTIONS; AXIAL COMPRESSION AND PURE BENDING
VOLUME VI -	INTERACTION BEHAVIOR

The material presented in this first volume is primarily intended to provide an introductory background which places the significance of the subsequent volumes in a proper perspective.

SECTION 2

BUCKLING CRITERIA

In the solution of buckling problems, a number of different approaches may be taken. The two most commonly used techniques are the minimum energy method and the bifurcation concept. The former is based upon the theorem of minimum total potential energy which may be stated as follows:

> A conservative system is in a configuration of stable equilibrium if, and only if, the value of the total potential energy is a relative minimum.

To apply this theorem, one must formulate the total potential energy of the system, impose the mathematical artifice known as a virtual displacement, and examine the sign of the second-order energy changes (second variation). The second variation must be positive definite (positive regardless of the sign of the virtual displacement) for stability to exist. The value of the applied load at which the second variation first ceases to be positive definite is the critical load for the system.

The bifurcation concept originally developed by Poincaré [credited in Ref. 1] in 1883 constitutes an equilibrium approach to the problem of buckling. Any point at which a single equilibrium path branches into two or more equilibrium paths is known as a bifurcation point. An example of this phenomenon is shown in Figure 4. This figure depicts the equilibrium paths for a perfect isotropic circular cylinder subjected to axial compression. As a rule, the unbuckled configuration becomes unstable at a bifurcation point and the bifurcation approach to buckling analysis simply involves the mathematical search for these branching points. In conducting this search, one must study the character of the equilibrium behavior. As in the solution by Block, Card, and Mikulas [2], this study may be done with the

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Numbers in brackets [ ] in the text denote references listed in SECTION 5.

assistance of energy principles. However, such investigations should not be misconstrued as constituting a minimum energy approach. In the minimum energy method, a so-called second variation is examined. On the other hand,

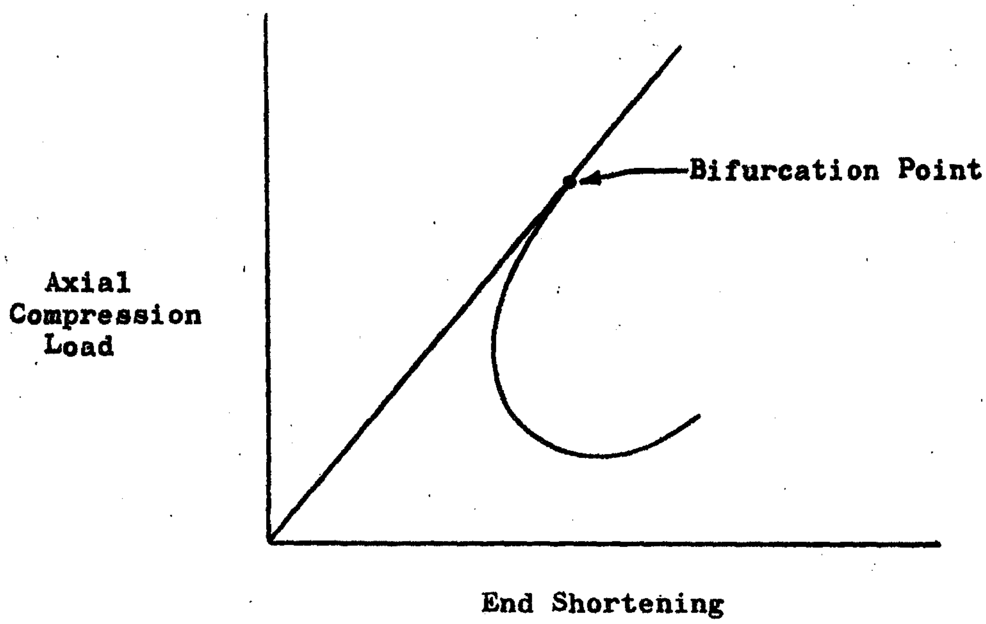

Figure 4 - Equilibrium Paths For a Perfect Isotropic Circular Cylinder Subjected to Axial Compression

the bifurcation method requires a study only of the so-called first variation. That is, in this case the system is subjected to a virtual displacement and the first-order change (first variation) in the total potential energy is tested for compliance with the principle of stationary potential energy which may be stated as follows:

> A necessary and sufficient condition for the equilibrium of an elastic body is that the first-order change in the total potential energy of the body be equal to zero for any virtual displacement.

Although this principle is often used in the application of bifurcation theory, it can sometimes lead to a loss of feel for the physical significance of the mathematical operations. Hence, many applications of the bifurcation concept are achieved without the use of energy methods. Instead, the basic laws of elementary mechanics are used to write the governing equilibrium equations for a distorted free-body element. To accomplish this, it is only necessary to perform simple summations of forces and moments in the several appropriate directions. This, of course, is only the first of a series of rather complicated steps in the derivation of a final buckling equation but, once having embarked upon this approach, each of the subsequent operations likewise retains greater physical transparency than do the variational techniques.

SECTION 3

BASIC ORTHOTROPIC CYLINDER EQUATION

The basic orthotropic cylinder equation utilized in Volumes III [3] and IV [4] of this report was developed from bifurcation theory without the use of energy concepts. The final equation may be written as follows:

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\,\beta^4\right]}{4\alpha\beta^4} + \frac{\alpha\,\beta^4\,(Z)^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \tag{3-1}$$

where,

$$Z = \left[1 - \frac{C_{11} + C_{22}}{2\alpha\,(A_{22}\,D_{22})^{1/2}\beta^2} - \frac{C_{12}}{2\alpha\,A_{22}\,(D_{22}/A_{11})^{1/2}} - \frac{C_{21}}{2\alpha\,(A_{11}D_{22})^{1/2}\beta^4}\right] \tag{3-2}$$

A detailed derivation of this relationship is given in reference 5 where the coordinate system shown in Figure 5 was used. It might be noted however that the final equation developed in reference 5 includes a quantity C_{33} which is a measure of the shear-center offset from the basic cylindrical skin. As written above, the fundamental equation is based on the assumption that the stringers and/or rings provide no resistance to γ_{xy} shear deformations so that $C_{33} = 0$. That is, it has been assumed that all of this in-surface type of shear restraint is furnished by the basic cylindrical skin. Furthermore, it has also been assumed that transverse shear strains γ_{xz} and γ_{yz} are everywhere zero. This, of course, is common practice in thin-shell theory. However, since this latter assumption is certainly not justified for most sandwich-type cylinders, equations (3-1) and (3-2) must be considered inapplicable to such configurations.

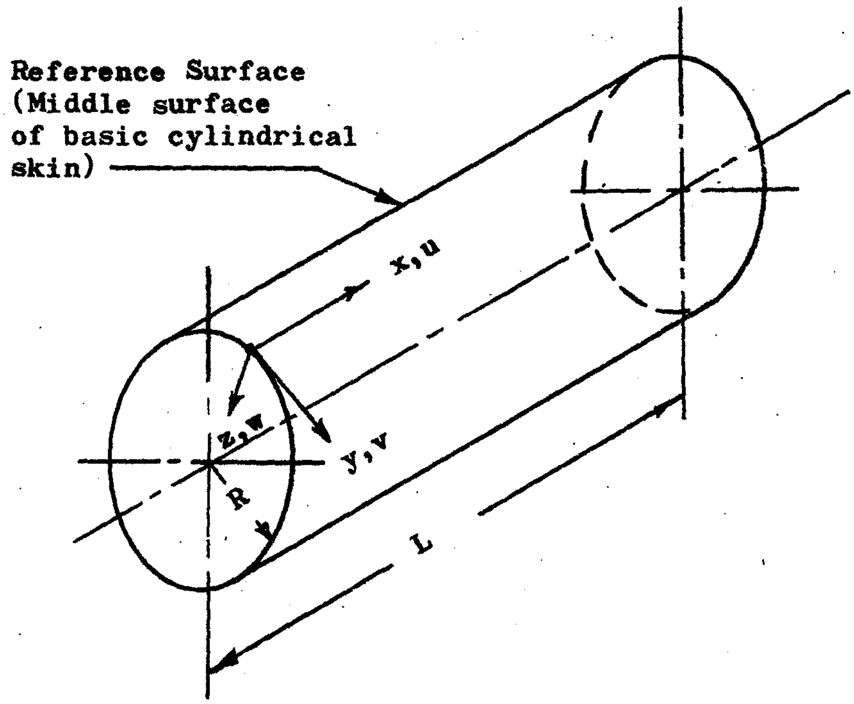

Figure 5 - Coordinate System

Equations (3-1) and (3-2) have been written in their rather compact, instructive forms through the incorporation of the following parameters, most of which were first proposed by Thielemann [6]:

$$\left(\overline{N}_{THIEL}\right)_c = -\overline{N}_{THIEL} = -\frac{\overline{N}_x R}{2}\left(\frac{A_{11}}{D_{22}}\right)^{1/2}$$

$$\eta_s = \frac{\left(A_{12} + \frac{A_{33}}{2}\right)}{\sqrt{A_{11}A_{22}}} \qquad (3\text{-}3)$$

$$\eta_p = \frac{\left(D_{12} + 2D_{33}\right)}{\sqrt{D_{11}D_{22}}}$$

$$\gamma = \frac{D_{11}A_{11}}{D_{22}A_{22}}$$

$$\beta = \left(\frac{m}{n}\right)\left(\frac{\pi R}{L}\right)\left(\frac{A_{22}}{A_{11}}\right)^{1/4} \qquad \text{(3-3) Cont'd.}$$

$$\alpha = \frac{L^2}{2Rm^2\pi^2 A_{22}\left(\frac{D_{22}}{A_{11}}\right)^{1/2}}$$

The various A_{ij}'s, D_{ij}'s, and C_{ij}'s of equations (3-2) and (3-3) are important fundamental constants which arise out of the following relationships between the stress resultants and the buckling distortions:

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} + C_{11}\overline{N}_x + C_{12}\overline{N}_y$$

$$M_y = -D_{21}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} + C_{21}\overline{N}_x + C_{22}\overline{N}_y$$

$$M_{xy} = -2D_{33}\frac{\partial^2 w}{\partial x \partial y} \qquad \text{(3-4)}$$

$$\epsilon_x = A_{11}\overline{N}_x + A_{12}\overline{N}_y + C_{11}\frac{\partial^2 w}{\partial x^2} + C_{21}\frac{\partial^2 w}{\partial y^2}$$

$$\epsilon_y = A_{21}\overline{N}_x + A_{22}\overline{N}_y + C_{12}\frac{\partial^2 w}{\partial x^2} + C_{22}\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = A_{33}\,N_{xy}$$

The minus signs in these equations are due to the particular sign conventions selected in the derivation. In addition it is pointed out that, in the above equations, stress resultants with bars above them are centroidal values. All other stress resultants are reference surface values. In

addition, all of the strain and displacement values are likewise measured in the reference surface. The A_{ij}'s and D_{ij}'s of equations (3-2) through (3-4) are usually identified as elastic constants while the C_{ij}'s might be suitably described by the terminology "eccentricity coupling constants".

It should be noted that equations (3-1) and (3-2) were derived for a monocoque orthotropic cylinder. However, these equations will be used in subsequent volumes [3,4] for the analysis of discretely stiffened shells. The key to success in these applications lies in the means employed to evaluate the elastic constants and the eccentricity coupling constants. From equations (3-4) it can be seen that these quantities are dependent upon the various structural rigidities of the shell wall. Practical formulas for computing these constants are presented in the procedures of Volumes III [3] and IV [4]. However, at this time it is profitable to devote some attention to their origins and to examine the formulations which would apply in two very special cases. In the first place, it is helpful to note that for an isotropic cylinder the elastic constants take on the following forms:

$$
\begin{aligned}
A_{11} &= A_{22} = \frac{1}{Et} \\
A_{12} &= A_{21} = -\frac{\nu}{Et} \\
A_{33} &= \frac{1}{Gt} \\
D_{11} &= D_{22} = \frac{Et^3}{12(1-\nu^2)} \\
D_{12} &= D_{21} = \frac{\nu Et^3}{12(1-\nu^2)} \\
D_{33} &= \frac{Gt^3}{12}
\end{aligned}
\qquad (3\text{-}5)
$$

As already noted, the equations of this volume are generally to be considered inapplicable to sandwich structures in view of the neglect of transverse shear strains (γ_{xz} and γ_{yz}) in the derivations. However, at this point it is still informative to note that the following formulas could be used to find the elastic constants in the very special case of a sandwich configuration having a core with infinite transverse shear rigidity:

$$\begin{aligned}
A_{11} &= A_{22} = \frac{1}{2t_f E} \\
A_{12} &= A_{21} = -\frac{\nu}{2t_f E} \\
A_{33} &= \frac{1}{2t_f G} \\
D_{11} &= D_{22} = \frac{E t_f h^2}{2(1-\nu^2)} \\
D_{12} &= D_{21} = \frac{\nu E t_f h^2}{2(1-\nu^2)} \\
D_{33} &= \frac{G t_f h^2}{2}
\end{aligned} \qquad (3\text{-}6)$$

where

t_f = Facing thickness
h = Distance between middle-surfaces of facings.

These equations are applicable when the facings are of the same material and of equal thickness and this thickness is small compared to h.

From equations (3-4), (3-5), and (3-6), it should be observed that

(a) A_{11} constitutes the reciprocal of the longitudinal extensional stiffness per unit length of circumference.

(b) A_{22} constitutes the reciprocal of the circumferential extensional stiffness per unit of axial length.

(c) D_{11} constitutes the longitudinal flexural stiffness per unit length of circumference.

(d) D_{22} constitutes the circumferential flexural stiffness per unit of axial length.

(e) A_{12} and A_{21} each constitute measures of coupling between extensional deformations in the longitudinal and circumferential directions.

(f) D_{12} and D_{21} each constitute measures of coupling between flexural deformations in the longitudinal and circumferential directions.

(g) A_{33} constitutes the reciprocal of the in-plane shear stiffness of the shell wall.

(h) D_{33} constitutes the twisting stiffness of the shell wall.

In addition it is pointed out that, for applications to skin-stringer-ring configurations, the constant C_{11} is simply the eccentricity (see GLOSSARY) of the effective skin-stringer combination. Correspondingly, C_{22} is simply the eccentricity of the effective skin-ring combination. The C_{12} and C_{21} values account for Poisson-ratio cross-linking of the eccentricity influences.

It is helpful to note that equations (3-1) and (3-2) essentially comprise an extension to the theory developed in reference 7 for longitudinally stiffened circular cylinders. The extension was accomplished in order to adapt the solution to circular cylinders having both stringers and rings. Equations (3-1) and (3-2) can be specialized to the case of a longitudinally stiffened circular cylinder by taking $C_{22} = C_{21} = 0$. This gives the result

$$\left(\bar{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\,\beta^4\right]}{4\alpha\beta^4} + \frac{\alpha\,\beta^4\,(Z)^2}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \qquad (3\text{-}7)$$

where

$$Z = \left[1 - \frac{C_{11}}{2\alpha\,(A_{22}\,D_{22})^{1/2}\,\beta^2} - \frac{C_{12}}{2\alpha\,A_{22}\,(D_{22}/A_{11})^{1/2}}\right] \qquad (3\text{-}8)$$

It is noted that, except for the C_{12} term, equations (3-7) and (3-8) are identical to equation (36) of reference 7 where the C_{12} term was evidently discarded as a negligible quantity for the particular test specimens of interest there. Further note that when $C_{11} = C_{22} = C_{12} = C_{21} = 0$, equations (3-1) and (3-2) reduce to the following:

$$\left(\overline{N}_{THIEL}\right)_c = \frac{\left[1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\beta^4\right]}{4\alpha\beta^4} + \frac{\alpha\beta^4}{\left[1 + 2\eta_s\beta^2 + \beta^4\right]} \qquad (3\text{-}9)$$

This expression is identical to that given by Almroth in references 8 and 9 except for an apparent typographical error involving the omission of a $\sqrt{\gamma}$ factor in one term of both references. In addition, it has also been established that equation (3-9) is identical to equation (3.2) in a report by Appel [10]. Equation (3-9) can be further simplified by ignoring the influences of finite cylinder length, in which case one can arrive at the small deflection equation of Thielemann [6] which may be expressed as follows:

$$\left(\overline{N}_{THIEL}\right)_c = \left[\frac{1 + 2\eta_p\sqrt{\gamma}\,\beta^2 + \gamma\beta^4}{1 + 2\eta_s\beta^2 + \beta^4}\right]^{1/2} \qquad (3\text{-}10)$$

Reference 11 gives a detailed presentation of the mathematical operations involved in this simplification.

To make proper use of equations (3-1) and (3-2), one must realize that these relationships simply establish the magnitudes of longitudinal compressive loads which will maintain an orthotropic cylinder in deflected configurations defined by the variables m and n. The quantity m is the number of axial half-waves in the buckle pattern while n is the related number of full circumferential waves. For any single combination of stiffness values, an infinite number of load-wavelength combinations can be

possible. The critical load is the lowermost load which is just sufficient to hold the shell in a non-cylindrical deflected shape. Therefore the computation of a critical load must involve a mathematical minimization. The nature of the problem is such that this minimization must be accomplished with respect to two independent wave-type parameters. The values m and n might be selected for this purpose although other choices, such as α and β, would be equally satisfactory.

Since the analysis techniques of Volumes III [3] and IV [4] are based upon the minimization of equations (3-1) and (3-2), these methods only give classical small-deflection solutions. That is, the critical load predictions are obtained by locating a lowermost bifurcation point along the initially linear equilibrium path of a perfect orthotropic cylinder. Use of this approach raises some important questions as to the influences from initial imperfections and their interrelationship with the shape of the postbuckling equilibrium path. This matter is taken up in detail in Volume V [12] of this report.

SECTION 4

OVERALL ANALYSIS PROCEDURE

The methods presented in the several volumes of this report provide practical means for the analysis of instability in axially compressed circular cylinders having eccentric stringers and/or rings. In order to actually apply these methods, one should proceed as follows:

(a) For cylinders having both stringers and rings, first compute the critical buckling stress for the curved isotropic skin panels which lie between the stiffeners. This value can be calculated by using the methods of Volume II [13]. These methods directly provide design values that have been reduced from classical results in order to account for the effects of initial imperfections. Therefore, no further knock-down factors need be applied to these particular stress values. Although buckling of the isotropic skin panels is usually not catastrophic, the related critical stress values must be established in order to compute effective skin widths for use in the investigation of other possible modes.

(b) Then determine the failing stresses for sections which lie between rings. For configurations which have no stringers, the monocoque cylinder curves given in the appendix of Volume IV [4] may be used for this purpose. These curves likewise provide design values which incorporate reductions from classical theory to account for the effects of initial imperfections. No further knock-down factors need be applied to these particular stress values. For configurations which include stringers, the methods of Volume III [3] should be used to obtain the failing stresses for sections lying between rings. It should be noted that these methods account for the possibility that crippling might occur. Note however that Volume III [3] only presents the results from classical theory and that, for design purposes, these values must

be reduced in accordance with the knock-down criteria of Volume V [12]. Whether or not the configuration includes stringers, at this point in the investigation it is assumed that the rings experience no radial displacement. The validity of this assumption will be tested in the step which follows below.

(c) By using the contents of Volume IV [4], the general instability (see GLOSSARY) stress for the overall stiffened cylinder must then be determined. Here again, the methods of Volume IV [4] only give classical values and, for design purposes, these values must be reduced in accordance with the knock-down criteria of Volume V [12].

(d) For design purposes, it is then assumed that catastrophic collapse of the structure will occur at the lower of the two values obtained from (b) and (c) above. That is, depending upon the particular geometric proportions, the failure can occur either by the panel instability (see GLOSSARY) or the general instability (see GLOSSARY) mode.

SECTION 5

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